Engineering Structures 196 (2019) 109300

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Measuring inherent structural damping of structure-TMD systems

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ARTICLE INFO

Keywords:
Inherent structural damping
Added effective damping
Structural monitoring
Tuned mass dampers (TMDs)
Random vibration

ABSTRACT

A practical method is proposed to enable the inherent structural damping of structures equipped with tuned mass dampers (TMDs) to be quantified. While the traditional random decrement technique cannot be applied to structure-TMD systems, a modified version is presented which accommodates the energy transfer between the structure and TMD. It is shown that cross-correlation functions can be used to determine the random decrement signatures of the structural and TMD responses. Linear regression is combined with an energy balance approach to estimate the inherent structural damping. The total effective damping of the structure is then estimated by using an existing method to predict the added effective damping provided by the TMD.

Numerical simulations reveal that the method predicts the inherent structural damping with acceptable accuracy if the duration of the measured response captures at least two thousand structural oscillation cycles, which corresponds to 3–6 h of data for most tall buildings. The method is most accurate when the inherent structural damping is relatively high, and the TMD mass ratio is relatively low. The method is applied to data collected from an anonymous super-tall building equipped with a TMD. The TMD is found to increase the effective damping of the tower from 1.0% to 3.0% of critical. These damping values are supported by measurements conducted when the TMD was briefly locked from moving, and also by theoretical predictions using the assessed dynamic properties of the structure and TMD.

1. Introduction

Tall buildings are often susceptible to excessive wind-induced motion, which can result in occupant discomfort and hastened deterioration of building enclosure components due to large interstorey drifts [1]. The susceptibility to motion is primarily a result of the building being lightweight, flexible, and lightly damped. Adding mass or stiffening the structure was traditionally employed to reduce building motions to acceptable limits [2]. However, these approaches increased construction costs, reduced the available floor area (due to larger structural elements), and generally increased the seismic loads that the building attracted in regions of high seismicity. Within the last few decades, supplementary damping systems - which are systems engineered to increase the rate of energy dissipation of the structure - have increased in popularity [3].

A dynamic vibration absorber (DVA) is a type of supplementary damping system that is often represented as an auxiliary spring-mass-dashpot system that is coupled to the primary structure. The tuned mass damper (TMD) and tuned sloshing damper (TSD, also known as the tuned liquid damper (TLD)) are two popular DVAs that have seen many real-world installations in recent years. When the structure experiences a resonant response due to wind loading, the auxiliary mass will begin to move out-of-phase with the structure. Although the focus of this study is TMDs, the findings of this paper may also be applied to TSDs, which are commonly represented as equivalent TMDs [4]. The TMD alters the mechanical admittance function of the coupled structure-TMD system. The TMD effectively increases the rate of energy dissipation of the structure-TMD system, which reduces the structure's response. Alternatively, the TMD can be interpreted as a device that passively produces a force that opposes the external excitation applied to the structure.

A TMD is typically installed near the top of the building, where the maximum modal displacement occurs for the building vibration mode being targeted for supplemental damping. A larger TMD effective mass relative to the generalized mass of the structure (the TMD mass ratio) will produce a greater motion reduction; however, in practice most TMD mass ratios are within the range of 0.5–5%. Optimal formulae can be used to determine the optimal TMD frequency and TMD damping ratio that will produce the greatest motion reduction for a given TMD mass ratio [5,6].

Although TMDs have been studied extensively in the literature, only a few studies have evaluated their full scale performance [7]. Since

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https://doi.org/10.1016/j.engstruct.2019.109300
Received 19 January 2019; Received in revised form 6 May 2019; Accepted 11 June 2019

dozens of TMDs are currently installed in buildings worldwide, it is important that their performance is verified and published to provide compelling full-scale evidence of their efficacy. However, the performance verification of full-scale structure-TMD systems is typically challenging.

The performance of a TMD is often measured using the concept of effective damping (sometimes called "equivalent" damping). The effective damping is the damping that the structure without the TMD would need to have to experience the same average level of response as the structure equipped with the TMD. Mathematically,

$$\zeta_{eff} = \zeta_S \frac{\sigma_X^2}{\sigma_{X-damped}^2} \tag{1}$$

where ζ_{eff} is the effective damping, ζ_S is the inherent structural damping, and σ_X^2 and $\sigma_{X-damped}^2$ are the variances of the structural displacement response without and with the TMD, respectively. Eq. (1) cannot be directly applied since the response of the building with and without the TMD cannot be measured simultaneously. Alternatively, the measurements could be conducted at different times, but only if the loading applied to the building is stationary through both sets of measurements. This stationarity requirement could only be ensured if the building was excited manually by a known and controlled force, which is challenging to do for a full-scale structure. Instead, ambient wind loading must typically be employed as an unknown, uncontrolled, and unmeasurable excitation.

The effective damping can also be determined theoretically from the mechanical admittance function of the structure, $H_s(\omega)$ [8]:

$$\zeta_{eff} = \frac{\pi}{4}\omega_s \left[\int_0^\infty |H_s(\omega)|^2 d\omega\right]^{-1} \tag{2}$$

where ω_s is the natural angular frequency of the structure. The challenge is then to determine the mechanical admittance function from the measurement data, which is also difficult to achieve without knowledge of the input excitation. Instead, a system identification must be performed whereby the dynamic properties (mass, frequency, and damping) of the structure and TMD are determined, and the mechanical admittance function is then generated theoretically. However, the accurate identification of the dynamic properties of a coupled structure-TMD system generally requires sophisticated signal processing algorithms [9].

Recently, a practical method has been proposed to directly quantify the added effective damping that is produced by a TMD [4]. The method was successfully employed to evaluate the performance of full-scale buildings equipped TSDs and TMDs [10]. The added effective damping is the difference between the effective damping and the inherent damping of the structure:

$$\zeta_{added} = \zeta_{eff} - \zeta_s \tag{3}$$

However, in Refs. [4] and [10] it was not possible to estimate the inherent structural damping of the building when it was coupled to the TMD or TSD. If the inherent structural damping was not known prior to the installation of the damping system, the total effective damping could not be verified. The inherent structural damping is particularly important for buildings equipped with TMDs that have small mass ratios, since the inherent structural damping is typically quite significant relative to the added effective damping provided by the TMD.

The random decrement technique has been a popular method to estimate the inherent damping of a single degree of freedom system subjected to random ambient excitation. The random decrement technique generates a random decrement signature by superposing a large number of segments of the response after a predefined trigger condition is achieved [11,12]. The method assumes that the component of the response due to the random loading will approach zero if many segments are averaged together, leaving only the free decay response from the selected trigger level. The damping can then be determined from the random decrement signature using the logarithmic decrement approach or another suitable form of parameter/system identification. Application of the traditional random decrement technique to a structure-TMD system is not possible since it is a highly-coupled multiple degree of freedom system with different underlying response characteristics. Nonetheless, the authors have seen incorrect usage of the method by practitioners, who fail to understand their violation of the requisite assumptions. A multi-modal random decrement technique has been proposed by Tamura et al. [13,14], and has been employed to identify the frequencies and damping of structures with closely spaced modes. The technique has been employed to predict dynamic properties when the measured response is composed of multiple uncoupled single-degree-of-freedom oscillators. However, it does not directly accommodate the strong coupling (energy transfer) that occurs between the structure and TMD, and is therefore not appropriate for structure-TMD systems with nonproportional damping.

This study presents a method to estimate the inherent structural damping of the combined structure-TMD system, which can then be used to determine the total effective damping of the structure-TMD system. First, the random decrement technique is reviewed, and it is shown why the traditional method cannot be applied to a coupled structure-TMD system. Random decrement signatures of the structure and TMD are derived using the cross-correlations of the structural and TMD responses. Using the random decrement signatures of the structure and TMD, an energy-balance approach is employed to estimate the inherent structural damping of the system. The method is then validated using numerical simulations. Lastly, the method is applied to full-scale monitoring data collected from an anonymous super-tall building equipped with a TMD system.

2. Structure-TMD system

The structure-TMD system is represented in Fig. 1, where M_S, C_S, and K_S are the generalized mass, damping, and stiffness of the structure (normalized to the location of TMD attachment), while m_{TMD}, c_{TMD}, and k_{TMD} are the equivalent mass, damping, and stiffness of the TMD, respectively. The corresponding equations of motion are:

$$(M_S + m_{TMD})\ddot{X} + m_{TMD}\ddot{x}_r + C_S\dot{X} + K_S X = F_{exc} \tag{4}$$

$$m_{TMD}\ddot{x}_r + c_{TMD}\dot{x}_r + k_{TMD}x_r = -m_{TMD}\ddot{X} \tag{5}$$

Where X is the generalized displacement of the structure, x_r is the relative displacement of the TMD, F_{exc} is the external applied wind loading, and a dot above a variable represents a time derivative. The total (absolute) displacement of the TMD is $x_{TMD} = X + x_r$. The typical dynamic quantities of the natural angular frequencies, damping ratios, and structure-TMD mass ratio are introduced:

$$\omega_S = \sqrt{\frac{K_S}{M_S}},\quad \omega_{TMD} = \sqrt{\frac{k_{TMD}}{m_{TMD}}} \tag{6}$$

$$\zeta_S = \frac{C_S}{2\omega_S M_S}\zeta_{TMD} = \frac{c_{TMD}}{2\omega_{TMD}m_{TMD}} \tag{7}$$

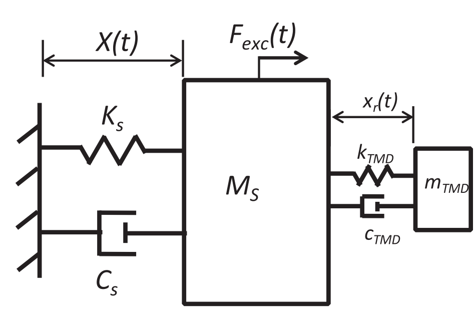

Fig. 1. Schematic of structure-TMD system representation.

$$\mu = \frac{m_{TMD}}{M_S} \tag{8}$$

Eq. (4) can be rearranged to be

$$\ddot{X} + 2\omega_S\zeta_S\dot{X} + \omega_S^2X = F_{exc}/M_S - \mu\ddot{x}_{TMD} \tag{9}$$

From Eq. (9), the structure can be considered a single degree of freedom system that is subjected to external wind loading, as well as the inertial forces from the TMD.

3. Random decrement signature

3.1. Single degree of freedom

For a single degree of freedom structure subjected to an excitation, $F_{exc}(t)$, the response of the structure after an initial trigger level ($X(0) = X_0$, $\dot{X}(0) = 0$) is the free vibration response of the structure plus Duhamel's convolution integral accounting for the continuing random excitation [15]:

$$X(t) = X_0 \exp(-\zeta_S\omega_S t)\cos(\omega_S t) + \int_0^t h(t-\tau)F_{exc}(\tau)d\tau \tag{10}$$

where $h(t)$ is the impulse response function of the single degree of freedom oscillator,

$$h(t) = \frac{1}{\omega_S M_S}\exp(-\zeta_S\omega_S t)\sin(\omega_S t) \tag{11}$$

The conditional expected value of Eq. (10) is then computed,

$$E[X(t)|X_0] = X_0 \exp(-\zeta_S\omega_S t)\cos(\omega_S t) + \int_0^t h(t-\tau)E[F_{exc}(\tau)|X_0]d\tau \tag{12}$$

where $E[Y(t)|X_0]$ is the expected value of $Y(t)$ given the initial condition $X(0) = X_0$. The random decrement signature is given by:

$$D_X(t) = E[X(t)|X_0] \tag{13}$$

It has been shown that if $F_{exc}(t)$ is stationary white noise, then $E[F_{exc}(\tau)|X_0] = 0$, which ensures the random decrement signature is equal to the free vibration response of the system from an initial amplitude of X_0 [15]. It is then straightforward to determine the inherent structural damping from this free decay response.

3.2. Structure-TMD system

For a structure-TMD system, Eq. (9) reveals that the structure must be analyzed as a single degree of freedom system that is subjected to an excitation, $F(t)$ composed of the white noise external excitation, $F_{exc}(t)$ as well as the force from the TMD, $m_{TMD}\ddot{x}_{TMD}(t)$:

$$F(t) = F_{exc}(t) - m_{TMD}\ddot{x}_{TMD}(t) \tag{14}$$

Substituting Eq. (14) into Eq. (12), and recognizing that $E[F_{exc}(\tau)|X_0] = 0$, produces:

$$E[X(t)|X_0] = X_0 \exp(-\zeta_S\omega_S t)\cos(\omega_S t) - m_{TMD}\int_0^t h(t-\tau)E[\ddot{x}_{TMD}(\tau)|X_0]d\tau \tag{15}$$

which can be expressed as

$$D_X(t) = X_0 \exp(-\zeta_S\omega_S t)\cos(\omega_S t) - m_{TMD}\int_0^t h(t-\tau)D_{\ddot{x}}(\tau)d\tau \tag{16}$$

$$D_{\ddot{x}}(t) = E[\ddot{x}_{TMD}(t)|X_0] \tag{17}$$

where $D\ddot{x}(t)$ is the random decrement signature of the TMD acceleration. The random decrement signature of the structural displacement consists of the free vibration response of the structure from an initial amplitude of X_0, plus the response due to the TMD loading. Therefore, if the random decrement signature of the TMD acceleration is known, the random decrement signature of the structure may be treated as a forced vibration problem. It is therefore necessary to find an efficient method to determine the random decrement signatures of the structural and TMD responses.

3.3. Random decrement signatures and correlation

In this section, it will be shown that the random decrement signatures are readily determined from the cross-correlations of the system response variables. The derivation procedure is similar to that employed by Vandiver et al. [15] to determine the traditional random decrement signature for a single degree of freedom system. The random decrement signature of a response variable, $Y(t)$ given a trigger variable, $Z(t)$ is defined as

$$D_Y(t) = E[Y_2|Z_1] = \int_{-\infty}^{\infty} Y_2 p(Y_2|Z_1)dY_2 \tag{18}$$

where $Y_2 = Y(t_2)$, and $Z_1 = Z(t_1)$. $Y(t)$ and $Z(t)$ could correspond to any structural or TMD response quantity (displacement, velocity, or acceleration), but it is assumed herein that the mean has been removed from the response. The cross-correlation of Z_1 and Y_2 is defined as:

$$R_{ZY}(t_1, t_2) = E[Z_1Y_2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Z_1Y_2p(Z_1, Y_2)dZ_1dY_2 \tag{19}$$

where $p(Z_1,Y_2)$ is the joint probability density function of Z_1 and Y_2, which may be expressed as $p(Z_1,Y_2) = p(Y_2|Z_1)p(Z_1)$, where $p(Y_2|Z_1)$ is the probability of Y_2 given Z_1. Substituting this expression into Eq. (19) yields,

$$R_{ZY}(t_1, t_2) = \int_{-\infty}^{\infty} Z_1p(Z_1)\left[\int_{-\infty}^{\infty} Y_2p(Y_2|Z_1)dY_2\right]dZ_1 \tag{20}$$

It is recognized that the integral within square brackets in Eq. (20) is actually Eq. (18). Therefore, if the response is stationary, and $\tau = t_2 - t_1$, Eq. (20) can be expressed as

$$R_{ZY}(\tau) = \int_{-\infty}^{\infty} Z_1p(Z_1)D_Y(\tau)dZ_1 \tag{21}$$

If Z_1 and Y_2 are jointly Gaussian random variables, then:

$$p(Z_1) = \frac{1}{\sqrt{2\pi}\sigma_Z}\exp\left(-\frac{Z_1^2}{2\sigma_Z^2}\right) \tag{22}$$

$$p(Z_1, Y_2) = \frac{1}{2\pi\sigma_Z\sigma_Y\sqrt{1-\rho(\tau)^2}}\exp\left(\frac{-1}{2\sqrt{1-\rho(\tau)^2}}\left(\frac{Z_1^2}{2\sigma_Z^2} + \frac{Y_2^2}{2\sigma_Y^2} - 2\rho(\tau)\frac{Z_1Y_2}{\sigma_Z\sigma_Y}\right)\right) \tag{23}$$

$$\rho(\tau) = \frac{R_{ZY}(\tau)}{\sigma_Z\sigma_Y} \tag{24}$$

where σ_Z and σ_Y are the standard deviations of Z and Y, and ρ is the correlation coefficient of Z and Y. Given $p(Y_2|Z_1) = p(Z_1,Y_2)/p(Z_1)$, and after some simple algebraic manipulation,

$$p(Y_2|Z_1) = \frac{1}{\sqrt{2\pi}\sigma_a}\exp\left(\frac{-1}{2\sigma_a^2}\left[Y_2 - \frac{R_{ZY}(\tau)}{\sigma_Z^2}Z_1\right]^2\right) \tag{25}$$

$$\sigma_a^2 = \sigma_Y^2\left(1 - \frac{R_{ZY}^2(\tau)}{\sigma_Z^2\sigma_Y^2}\right) \tag{26}$$

Finally, substituting Eq. (25) into Eq. (18) and evaluating, and setting $Z_1 = Z_0$, produces

$$D_Y(\tau) = \frac{R_{ZY}(\tau)}{\sigma_Z^2}Z_0 \tag{27}$$

Eq. (27) indicates that the random decrement signature of a response quantity (Y) from an initial trigger level Z_0 is equal to the cross-correlation of the two responses normalized by the variance of Z, and multiplied by the trigger level, Z_0. It should be emphasized that the response and trigger variables (Y and Z) could be any structural or TMD response quantity. However, when generating the random decrement signatures of any response variable, the same trigger variable, Z, must be used for all random decrement signatures. For this study, the structural acceleration was used as the trigger variable for all calculations (that is, $Z = \ddot{X}$).

With the random decrement signatures of the structural displacement, $D_X(t)$ and TMD acceleration, $D\ddot{x}(t)$ determined, Eq. (16) could be employed to find the inherent structural damping, ζ_S. However, Eq. (16) is difficult to evaluate due to the nature of the convolution integral. It will be shown that a simpler approach is often to consider the energy of the system.

3.4. System energy

The inherent structural damping can be determined from the random decrement signatures of the structure and TMD by tracking the energy transfer within the system. The total energy of the structure can only change if energy is transferred to or from the structure by the TMD, or removed from the structure by its inherent structural damping. Recall that in the generation of the random decrement signatures, the effects of the external wind loading are removed. The total energy of the structure, $E_S(\tau)$ can be determined from the random decrement signatures of the structural displacement and velocity according to

$$E_S(\tau) = \frac{1}{2}M_S D_{\dot{X}}^2(\tau) + \frac{1}{2}\omega_S^2 M_S D_X^2(\tau) \tag{28}$$

The rate of energy addition or removal is the power acting on the structure, $P(\tau)$:

$$P(\tau) = (m_{DVA} D_{\ddot{x}}(\tau) + 2\omega_S\zeta_S M_S D_{\dot{X}}(\tau)) D_{\dot{X}}(\tau) \tag{29}$$

The total energy of the structure may therefore be expressed as:

$$E_S(\tau) = E_S(0) + \int_0^\tau P(t)dt \tag{30}$$

where $E_S(0)$ is the energy of the structure at the beginning of the random decrement signature. Combining Eqs. (29) and (30),

$$E_S(\tau) = E_S(0) + m_{TMD}\int_0^\tau D_{\ddot{x}}(t) D_{\dot{X}}(t)dt + 2\omega_S\zeta_S M_S \int_0^\tau D_{\dot{X}}^2(t)dt \tag{31}$$

Note that m_{TMD} and M_S are known with reasonable accuracy, and ω_S can be estimated from the measurements. Since the random decrement signatures of the response quantities were determined in the previous section, the only unknown in Eq. (31) is the inherent structural damping, ζ_S. The solution is quite sensitive to the initial energy of the structure, $E_S(0)$; small errors in $E_S(0)$ may result in significant changes to the predicted inherent structural damping. Therefore, $E_S(0)$ is treated as an unknown, and a least squares regression is performed to predict $E_S(0)$ and ζ_S. Rearranging Eq. (31) and setting

$$y_i = \zeta_S x_i + E_S(0) \tag{32}$$

where

$$y_i = E_S(\tau_i) - m_{TMD}\int_0^{\tau_i} D_{\ddot{x}}(t) D_{\dot{X}}(t)dt \tag{33}$$

$$x_i = 2\omega_S M_S \int_0^{\tau_i} D_{\dot{X}}^2(t)dt \tag{34}$$

enables least squares linear regression to be used to yield

$$\zeta_S = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2} \tag{35}$$

$$E_S(0) = \frac{1}{n}\sum y_i - \zeta_S \frac{1}{n}\sum x_i \tag{36}$$

In summary, the random decrement signatures are employed to determine the energy and energy transfer of the system. The difference between the energy input by the TMD and the structural energy must equal the energy removed by the inherent structural damping. Linear regression is employed to predict the inherent structural damping that produced the required energy dissipation. An intuitive check on the goodness of fit of the predicted inherent structural damping is to substitute it into Eq. (16) and regenerate $D_X(\tau)$. The measured random decrement signature of the structural displacement can then be compared to that regenerated using Eq. (16).

3.5. Total effective damping

The total effective damping of a building is equal to the sum of the inherent structural damping and the added effective damping that is provided by the TMD. It has been shown that the added effective damping can be estimated using the following formula [8]:

$$\zeta_{added} = \frac{\omega_S \mu E[\ddot{X}\dot{x}_r]}{2\sigma_{\ddot{X}}^2} \tag{37}$$

The added effective damping calculation requires knowledge of TMD mass ratio, the natural frequency of the structure, and the measured building acceleration and TMD relative velocity. This practical formula can be employed with the inherent structural damping calculation of the previous section to determine the total effective damping of the structure.

4. Simulations

4.1. System

A suite of numerical simulations of a structure-TMD system are conducted to evaluate the ability of the proposed method to estimate the inherent structural damping. The structure has a generalized mass of 1E7 kg, and a natural angular frequency of 1.26 rad/s. Various inherent structural damping ratios are assumed as part of this study. Linear TMDs with various mass, tuning, and damping ratios are evaluated. For each case considered, the structure is subjected to artificially-generated band-limited white noise force excitation (0.3–2.2 rad/s) for approximately 35 h of simulated time. The structure-TMD equations of motion given by Eqs. (4) and (5) are then solved in Matlab using the Runge-Kutta-Gill method. To study the rate of convergence, the method to determine the total effective damping proposed herein is evaluated for several durations of responses. The duration of the response is quantified as the number of structural oscillation cycles that would be completed, which is the product of the duration of the signal and the structural frequency.

TMDs with mass ratios of 0.5%, and 4.0% are considered, along with inherent structural damping ratios of 0.5%, 1.0%, and 2.0%. The values correspond to low and high TMD mass ratios, as well as low, moderate, and high levels of inherent structural damping. The TMDs are designed to have optimal tuning and damping, according to well-known optimal formulae for lightly-damped structures [5]. The properties of the linear TMDs considered are shown in Table 1.

4.2. Results

The random decrement signatures obtained when the TMD mass ratio is 0.5%, and the inherent structural damping is 1.0% are shown in Fig. 2. These random decrement signatures were computed by employing the full length of the simulation record. The time lag, τ is normalized by the period of the building. As expected, the structural displacement (Fig. 2(a)) and the structural velocity (Fig. 2(b)) are out-

Table 1
Summary of TMD properties.

μ	ω_{TMD}/ω_S	ζ_{TMD}
0.5%	99.6%	3.5%
1.0%	99.0%	5.0%
2.0%	98.5%	7.1%
4.0%	97.1%	9.8%

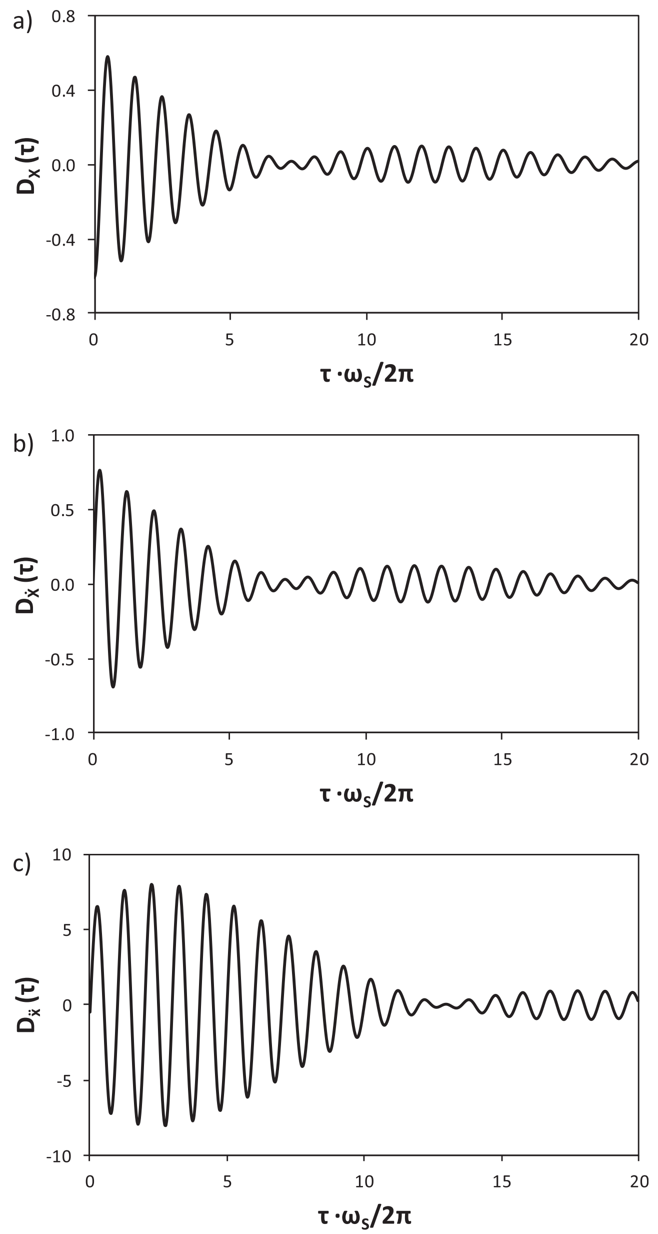

Fig. 2. Typical random decrement signatures of (a) structural displacement, (b) structural velocity, (c) TMD acceleration.

of-phase by one-quarter cycle. The TMD acceleration is approximately in-phase with the structural velocity (Fig. 2(c)), which means that it performs work on the structure.

The methodology presented in the proceeding sections was employed to predict the inherent damping of the structure. The predicted inherent structural damping is 0.98%, which is very close to the actual value of 1.0%. Fig. 3 shows the agreement that is obtained if Eq. (16) is employed to regenerate the random decrement signature of the structural displacement using the random decrement signature of the TMD acceleration. The agreement between the measured and the regenerated structural displacement random decrement signature is excellent, indicating the inherent structural damping estimate is accurate.

Fig. 4 shows the predicted and actual inherent structural damping, the added effective damping, and the total effective damping of the

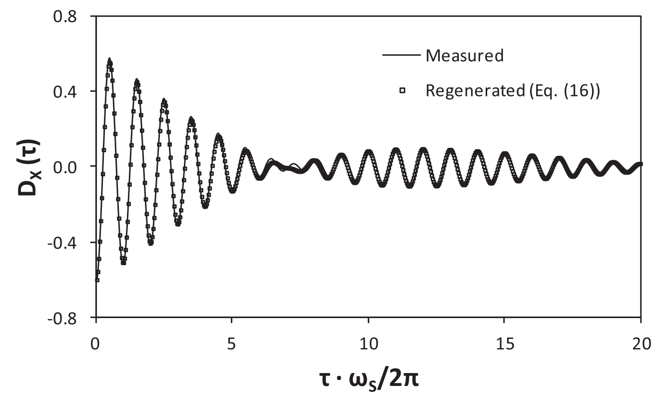

Fig. 3. Measured and regenerated random decrement signature of structural displacement.

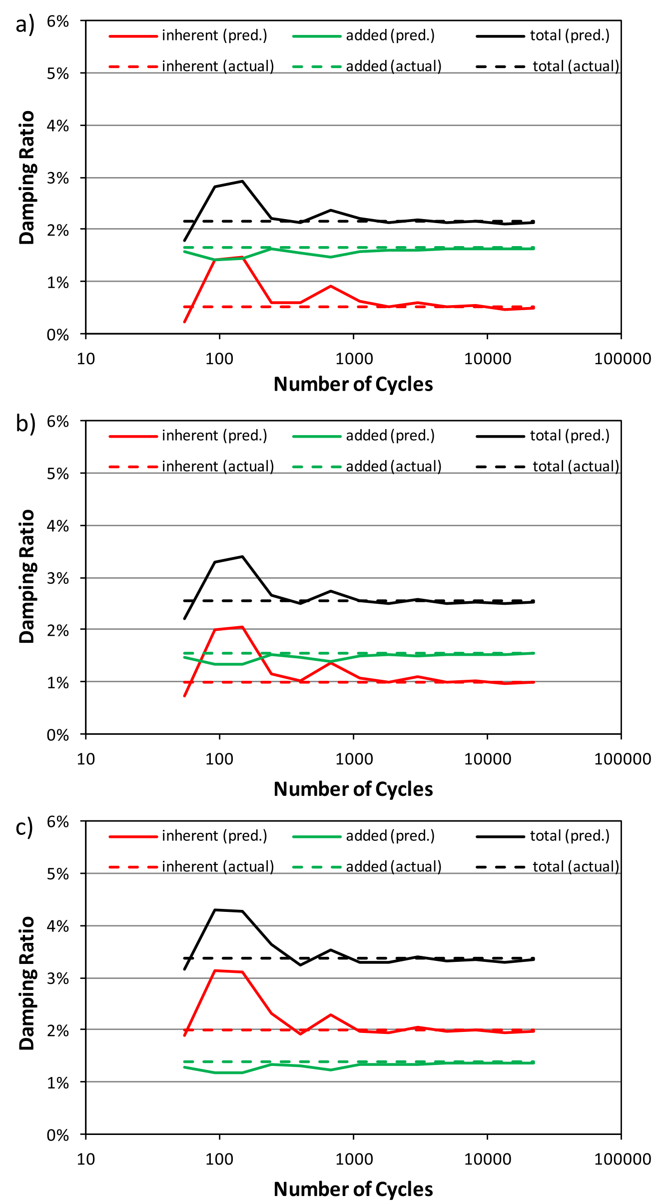

Fig. 4. Damping predictions vs number of structural oscillations ($\mu = 0.5\%$) (a) $\zeta_S = 0.5\%$, (b) $\zeta_S = 1.0\%$, (c) $\zeta_S = 2.0\%$.

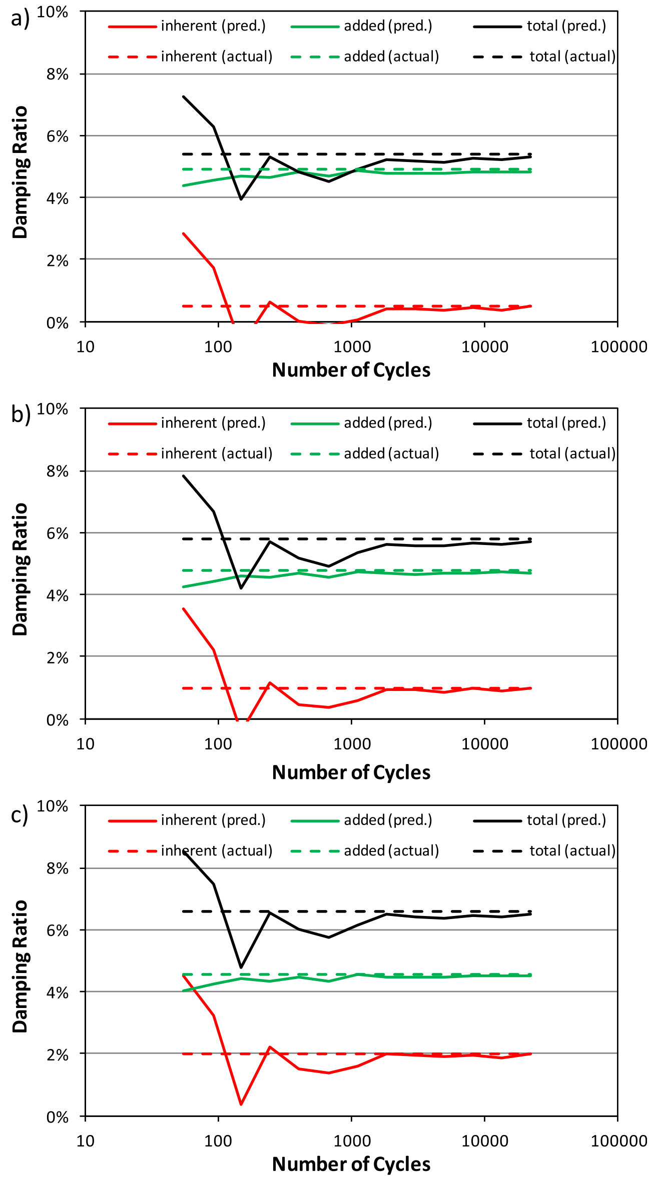

Fig. 5. Damping predictions vs number of structural oscillations (μ = 4.0%) (a) ζ_S = 0.5%, (b) ζ_S = 1.0%, (c) ζ_S = 2.0%.

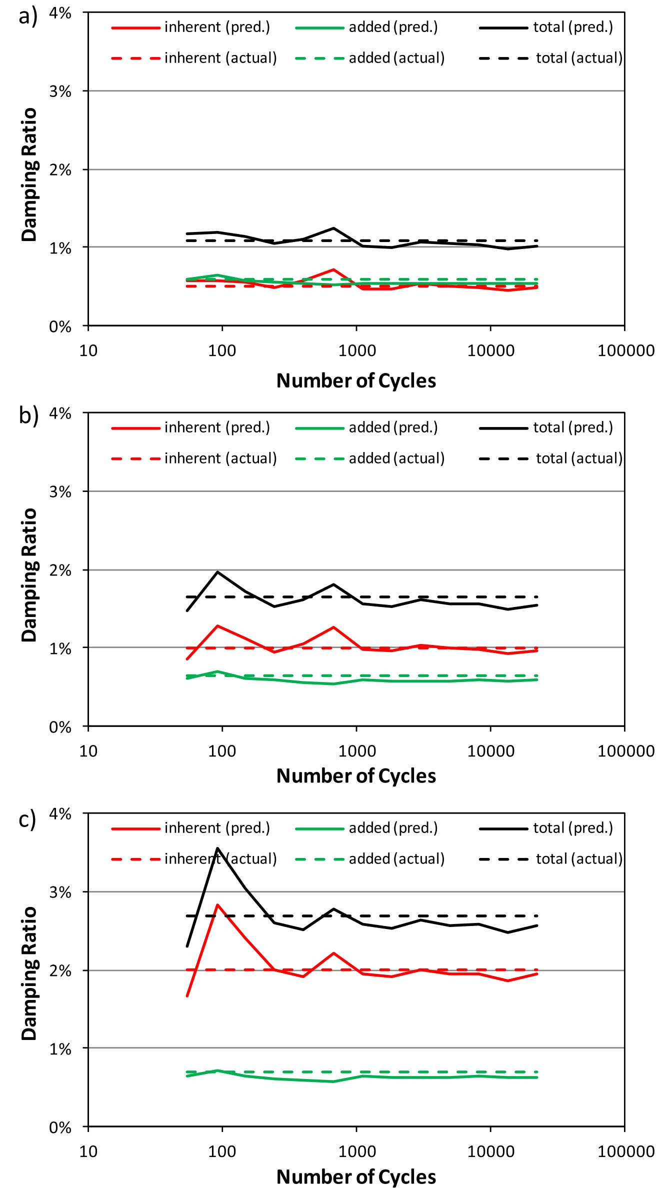

Fig. 6. Damping predictions vs number of structural oscillations - mistuned TMD (μ = 0.5%) (a) ζ_S = 0.5%, (b) ζ_S = 1.0%, (c) ζ_S = 2.0%.

structure-TMD system when the TMD mass ratio is 0.5%, and the inherent structural damping is 0.5%, 1.0%, and 2.0%. Similarly, Fig. 5 shows the predicted and actual damping levels when the TMD mass ratio is 4%, and the inherent structural damping is 0.5%, 1.0%, and 2.0%. In all cases, significant scatter is shown in the predicted inherent structural damping when the number of structural cycles is less than approximately 2000. Scatter is also shown in the predicted added effective damping when fewer than 2000 cycles are employed in the calculation, however, the scatter is far less pronounced than that in the predicted inherent structural damping. Obtaining a minimum of 2000 structural oscillation cycles of typical high-rise buildings (with periods between 5 and 10 s) would require between 3 and 6 h of data, which is expected to be a manageable duration for a monitoring program. Longer monitoring durations will provide more accurate damping estimates by reducing the errors associated with the cross-correlation estimates for the structure and TMD responses. While a detailed theoretical or statistical analysis of the rate of convergence of these estimates is worthy of investigation, it is beyond the current scope of work.

As the inherent structural damping increases, the relative error between the predicted and actual damping decreases, although the absolute error remains similar. This trend is expected because, by comparison, the energy input or output is less dominated by the TMD. In those systems where the TMD dominates the flow of energy in the system, the problem of finding the inherent structural damping is ill-posed, as the structural vibrational energy is only weakly sensitive to changes of the inherent structural damping. However, if the inherent structural damping is high, and/or the TMD mass ratio is low, then the flow of energy in the system is dominated by the inherent structural damping, making predictions of that damping more accurate. The implication of this trend is that if the inherent structural damping is highly important to the calculation of the total effective damping, then its prediction is also expected to be more accurate.

The ability of the proposed method to predict the system damping when the TMD is mistuned is investigated. A TMD with a mass ratio of 0.5%, a non-optimal frequency tuning ratio of 95%, and a TMD damping ratio of 1.5% is considered. Fig. 6 shows the predicted and actual inherent structural damping, added effective damping, and total effective damping levels. The proposed method provides a reasonable estimate of system damping when the number of structural cycles exceeds 2000. Thus, the method performed well when the TMD was not optimally tuned.

5. Application

In this section, the proposed methodology is applied to full-scale monitoring data of an anonymous super-tall building. The natural angular frequency of the building is 0.67 rad/s, and it is equipped with a TMD with a mass ratio equal to 1.0%. The TMD tuning ratio is slightly higher than optimal at 101.4%, and the TMD damping ratio is close to optimal at 4.8%. The slightly suboptimal tuning was a result of the building frequency decreasing during the final stages of construction; perhaps due to concrete cracking, or the addition of slightly more dead or live load than anticipated. However, it was determined that retuning the TMD would only improve the acceleration reduction from 43% to 45%, which was not considered worthwhile. The building and TMD have been monitored during several wind events that produced accelerations at the top of the building that were close to 1 milli-g. While these building accelerations are relatively small and imperceptible to occupants, the motion of the building and TMD were clearly observed in the data collected. The inherent structural damping and added effective damping will be predicted using the methodology proposed herein.

For this building, it was possible to mechanically "lock-out" the TMD (prevent its movement) for approximately 30 min during a wind event. With the TMD locked, the building will respond as a single degree of freedom structure (albeit with an increased generalized mass due to the locked TMD), which enables its inherent structural damping to be estimated using the traditional random decrement technique. Fig. 7 shows the random decrement signature generated for the building when the TMD was locked. By fitting a free decay envelope to the random decrement signature, the inherent structural damping ratio is estimated to be 1.0%. The peak building acceleration measured during this wind event was 0.8 milli-g.

The building was monitored for approximately 4.7 h during a separate wind event in which the TMD was operational. This duration of time corresponds to approximately 1800 cycles of motion. Fig. 8 shows a segment of the recorded structural and TMD accelerations, during which the peak building acceleration of 1.8 milli-g occurred and the peak TMD relative displacement was approximately 200 mm. Fig. 9 shows the random decrement signatures of the building dynamic displacement, velocity, and the TMD acceleration. The methodology proposed herein predicts an inherent structural damping of 0.96%, and an added effective damping of 2.07%, yielding a total effective damping of 3.03%. Fig. 10 shows that the random decrement signature of the structural displacement as measured and as regenerated using Eq. (16) are in good agreement. The predicted inherent structural damping is very close to that measured (1.0%) when the TMD was locked.

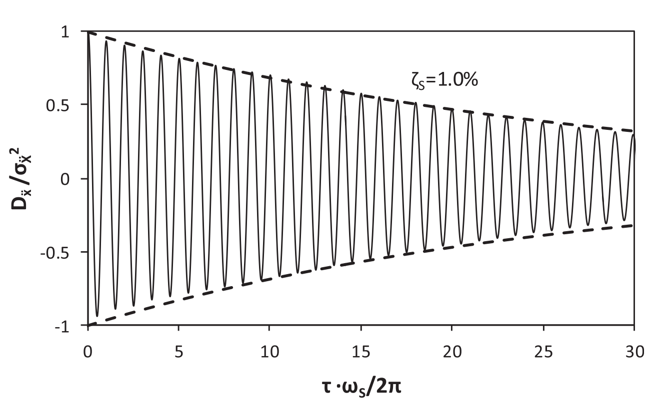

Fig. 7. Normalized random decrement signature of building when TMD is locked.

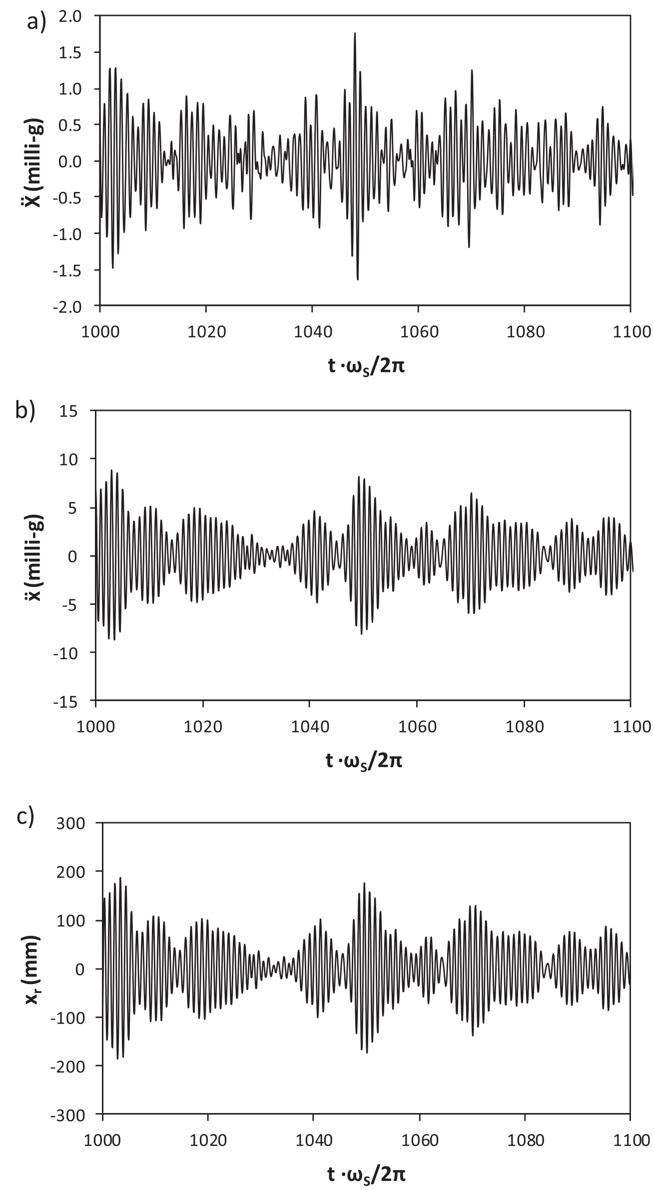

Fig. 8. Sample of (a) structural acceleration, (b) TMD acceleration, and (c) TMD relative displacement.

With the dynamic properties of the structure and TMD estimated, it is possible to theoretically predict the system response and thereby the TMD performance. Using the formulae derived by McNamara [16], the total effective damping is theoretically determined to be 3.09%, which is in very good agreement with the predictions of the proposed method. Using the spectra formulae found in McNamara [16], the acceleration response spectra of the structure and TMD are computed and shown in Fig. 11 plotted against the measured spectra. Some discrepancies can be observed between the peaks of the spectra, but the agreement between the modelled and full-scale monitoring data is reasonable.

The results of the full scale monitoring suggest that the effective damping of the structure has been increase from approximately 1.0% to 3.0%. This result has been predicted using the method proposed herein, as well as theoretical predictions using the identified properties of the structure and TMD. Using Eq. (1), the TMD will produce a 43% reduction of the structural acceleration.

6. Conclusions

A method has been proposed to identify the inherent structural

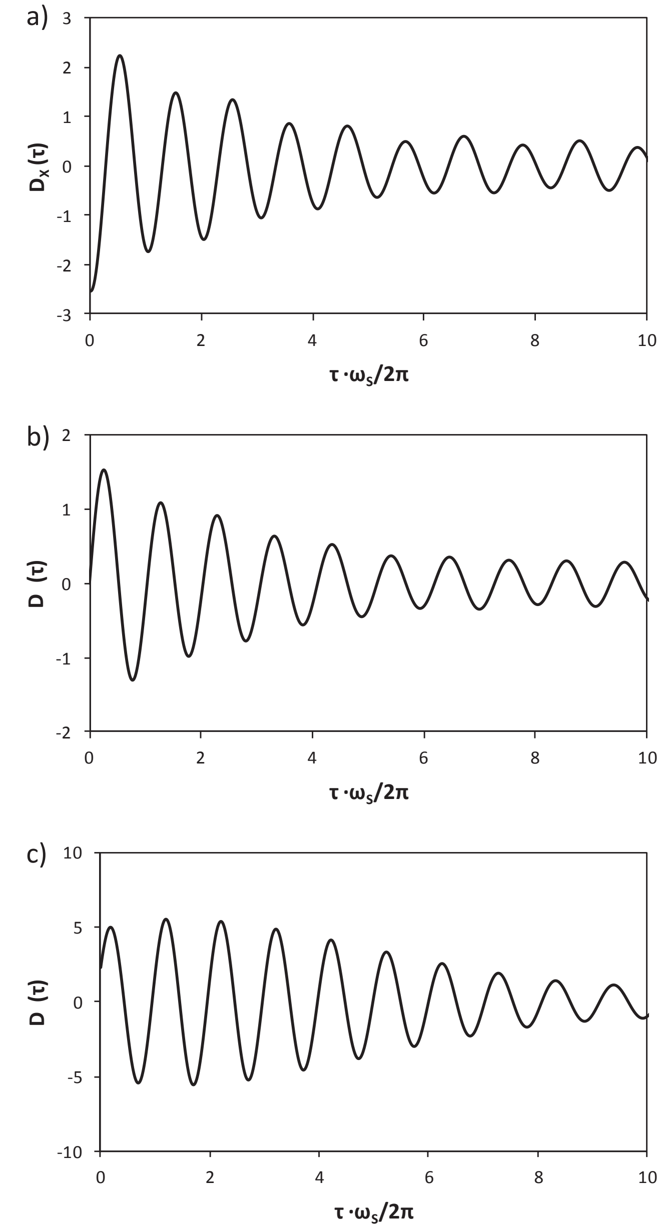

Fig. 9. Random decrement signatures of (a) structural displacement, (b) structural velocity, (c) TMD acceleration.

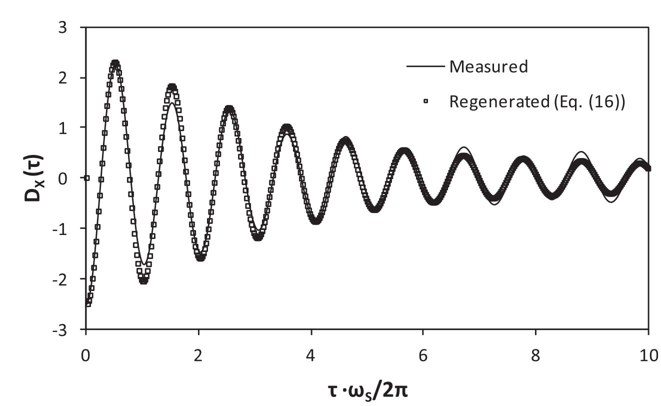

Fig. 10. Measured and regenerated random decrement signature of structural displacement.

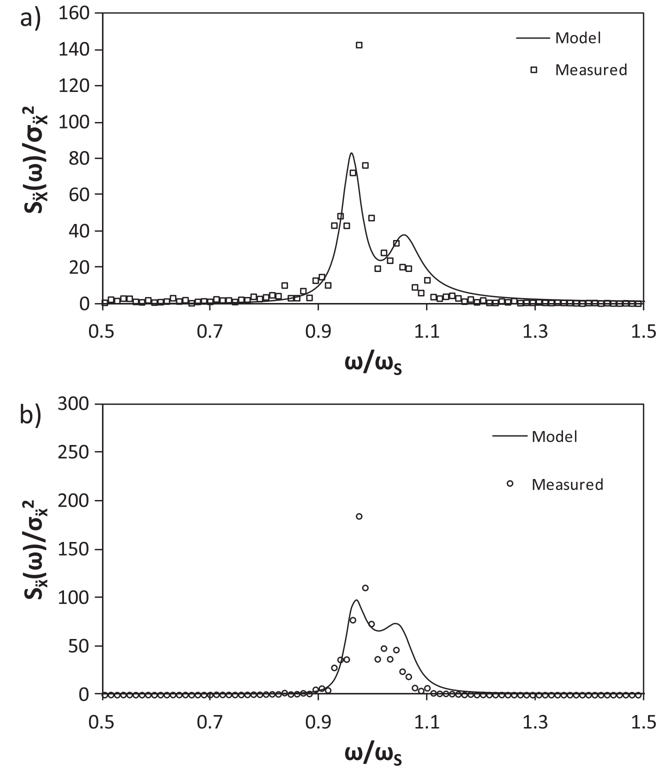

Fig. 11. Modelled and measured normalized acceleration spectra (a) structure, (b) TMD.

damping of structures equipped with tuned mass dampers (TMDs). The method may also be applied to tuned sloshing dampers (TSDs), which are often represented as equivalent TMDs. The method requires the response of the structure and TMD to be measured, and the generalized mass of the structure and TMD to be known. Random decrement signatures of the structural and TMD responses were determined using cross-correlation functions. An energy-based approach allows the inherent structural damping to be estimated using linear regression. When the proposed method is used in conjunction with an existing method to predict the added effective damping provided by a TMD, the total effective damping can be estimated from full-scale measurements.

Simulations of a structure-TMD system were conducted to evaluate the proposed model for various levels of inherent structural damping and TMD mass ratios. For the simulations conducted, the model predicted the inherent structural damping with excellent accuracy when the duration of the monitoring period was such that over 2000 cycles occurred.

Lastly, the model was employed to evaluate the performance of a TMD installed in an anonymous super-tall building. The proposed method indicated that the TMD increased the effective damping of the structure from 1.0% to 3.0%, which would produce a 43% decrease of the structural accelerations. The predicted inherent structural damping was verified using measurements conducted when the TMD was locked, enabling the traditional random decrement technique to be employed. The predicted added effective damping was in agreement with theoretical predictions made using the estimated dynamic properties of the structure and TMD.

The method proposed in this study will enable the performance of TMDs installed in full-scale structures to be more efficiently verified. Moreover, the results of the full-scale monitoring presented are further evidence that TMDs can be an effective means to reduce the wind-induced dynamic motion of structures.

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